Export-Platform FDI: Cannibalization or Complementarity?

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The dominant branch of the economics literature on multinational firms (MNEs) treats their final-good production location choices as substitutes. In these models, global firms face a 'proximity-concentration tradeoff' in which their plant-location decisions depend on the cost of production in each country, the size of trade costs between production and consumption locations, and the benefits of concentrating production in fewer locations to reduce fixed overhead costs (Markusen, 1984; Helpman et al., 2004; Tintelnot, 2017). In these settings, improvements in one country's productivity generate cannibalization effects that reduce the profitability of operating affiliates in other countries.

Recent empirical work, however, suggests that MNEs' plant locations may not always be substitutes. Garetto et al. (2019) find that US MNEs' affiliate sales in certain countries are *unaffected* by their affiliate activities in other countries. Using newly merged data on US firms' trade and multinational activity by country, Antràs et al. (forthcoming) show that US MNEs are not only more likely to export to countries in which they have affiliates, but also to other countries that are proximate to those affiliates, a fact that is hard to square with canonical 'export-platform' FDI models.

This paper provides conditions under which a model of export-platform FDI generates complementarities rather than cannibalization effects across MNEs' production locations. We first develop a baseline model similar to Tintelnot (2017) in which final goods are produced only with labor and there are no fixed costs to export. Perhaps surprisingly, this model does *not* necessarily generate cannibalization effects. We derive a simple condition that determines whether an MNE's plants are substitutes or complements. This condition is shaped by the relative size of (i) the *cross-firm* elasticity of demand the MNE faces for its goods and (ii) the *within-firm* elasticity of labor substitution across the MNE's plants.

Having developed this baseline model, we introduce destination-specific fixed costs of exporting that are incurred at the *firm level*, and show that this extension expands the range of parameter values for which the model delivers complementarity across MNEs' production locations. Finally, we introduce tradable intermediate inputs and show that whenever global sourcing entails firm-by-country-specific fixed costs of sourcing shared across all the MNE's plants, the range of parameter values for which assembly location decisions are complements is again expanded.

I. A Model of Export-Platform FDI

We begin by developing a simple multi-country model of export-platform FDI similar to Tintelnot (2017).

A. Environment and Preferences

We consider a world in which individuals in J countries consume differentiated manufactured goods produced by heterogeneous firms using labor. Although each firm produces a single good, we assume that this firm's good is differentiated based on its production country and that the same firm may produce in multiple countries.

We index firms by φ and varieties within firms by k. Given our Armington assumption, k also corresponds to an index for production locations. We assume a nested CES structure in which the degree of substitutability across varieties produced by different firms may differ from the degree

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of substitutability across varieties produced by different plants of the same firm. More formally, preferences are represented by

$$(1) \qquad U_{Mi} = \left(\int_{\varphi \in \Omega_i} \boldsymbol{q}_i(\varphi)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\varphi \right)^{\frac{\sigma}{\sigma-1}},$$

where Ω_i is the endogenous measure of firms selling differentiated goods in country i, and where the firm-specific composite $\boldsymbol{q}_i(\varphi)$ is

$$(2) \qquad \boldsymbol{q}_i(\varphi) = \left(\sum_{k \in \mathcal{K}(\varphi)} q_i(\varphi, k)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The set $\mathcal{K}(\varphi) \subseteq J$ includes the assembly locations from which firm φ sells varieties.

The parameter σ governs the *cross-firm* elasticity of demand for a firm's bundle of products. Our Armington assumption introduces an additional parameter ε controlling *within-firm* factor substitutability across a firm's various active plants. Because labor is the only factor of production, ε also governs the elasticity of labor substitution within an MNE. In section II, we extend our results to a more general setting, which encompasses cases with *no* product differentiation by country of production.

It is straightforward to show that the above preferences imply that consumers in country i spend an amount

$$(3) \qquad S_{ki}(\varphi) = \left(\frac{p_i(\varphi, k)}{\boldsymbol{p}_i(\varphi)} \right)^{1-\varepsilon} \times \left(\frac{\boldsymbol{p}_i(\varphi)}{P_i} \right)^{1-\sigma} E_i$$

of their income on variety k produced by firm φ. In this expression, $p_i(\varphi, k)$ is the price charged for that variety k, $\boldsymbol{p}_i(\varphi)$ is the overall price index for varieties sold by firm φ, and P_i is the economy-wide ideal price index in country i (see Appendix A for formal definitions). E_i is total spending on manufactured goods in country $i \in J$.

B. *Manufacturing Production*

Manufactured varieties are produced under increasing returns to scale and monopolistic competition. The variable φ used to index final-good firms also corresponds to their 'core' productivity, which firms only learn after incurring a fixed cost of entry.

After paying this fixed cost, each firm acquires blueprints to produce varieties of a final good. Although the firm could produce its varieties anywhere in the world, we assume that opening an assembly plant in a given country $k \in J$ entails a fixed overhead cost equal to f_k^a units of labor in country k. In equilibrium, firms therefore open a limited number of assembly plants (possibly a single one). We denote the optimal set of countries $k \in J$ for which firm φ has paid the associated fixed cost of assembly by $\mathcal{K}(\varphi) \subseteq J$, and refer to it as the firm's *global assembly strategy.*

For the time being, we assume that production of final-good varieties requires only local labor. The cost at which a firm can manufacture in each location k is shaped by its core productivity φ, by firm-location specific wages w_k (which the firm takes as given), and by a firm-location-specific productivity parameter Z_k^a. Shipping final goods from country k to country i also entails variable (iceberg) trade costs τ_{ki}^a. For now, we abstract from fixed costs of exporting.

C. *Interdependencies in the Intensive Margin*

The model delivers a simple, closed-form solution for sales of an assembly plant in k to each market i (see Appendix A):

$$(4) \qquad S_{ki}(\varphi) = \kappa \varphi^{\sigma-1} \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon} \times \left(\Psi_i(\varphi)\right)^{\frac{\sigma-\varepsilon}{\varepsilon-1}} P_i^{\sigma-1} E_i,$$

where κ is a constant, $\xi_k^a \equiv \left(w_k / Z_k^a\right)^{1-\varepsilon}$ captures plant k's *assembly potential*, and Ψ_i is given by

$$(5) \qquad \Psi_i(\varphi) \equiv \sum_{k' \in J} \mathcal{I}_{k'}^a \cdot \xi_{k'}^a \left(\tau_{k'i}^a\right)^{1-\varepsilon},$$

with $\mathcal{I}_{k'}^a$ taking a value of 1 when $k' \in \mathcal{K}(\varphi)$, and a value of zero otherwise. Although both $\mathcal{I}_k^a$ and ξ_k^a are firm-specific variables, we omit their dependence on φ to make the notation less cumbersome.

Holding the firm's global assembly strategy fixed, equation (4) indicates that an idiosyncratic increase in plant k's assembly potential ξ_k^a naturally raises sales of this plant k to all countries $i \in J$.

Whether changes in ξ_k^a generate positive or negative effects on the sales to country i of plants based in *other* countries $k' \neq k$ is less clear-cut, and depends on the relative size of σ and ε. When the cross-firm elasticity of demand for goods is low relative to the degree of within-firm labor substitution across plants (i.e., $\sigma < \varepsilon$), cannibalization effects dominate and the sales of a particular plant k' of firm φ are decreased by efficiency improvements in its other plants. Conversely, when the cross-firm demand elasticity is high relative to the degree of within-firm labor substitution across plants ($\sigma > \varepsilon$), complementarity effects dominate and improvements in plant k's efficiency in serving market i also increase the sales of other plants k' in i.

Intuitively, a lower price of variety k' reduces the share of sales of plant k, but it also reduces the firm-level price index $\boldsymbol{p}_i(\varphi)$ and this shifts spending away from other firms and toward the goods produced by firm φ in all of its locations. If consumers' price sensitivity to $\boldsymbol{p}_i(\varphi)$ is greater than the elasticity of labor substitution within the firm ($\sigma > \varepsilon$), the latter effect dominates the former.

D. *Interdependencies in the Extensive Margin*

We now analyze the optimal set of countries in which a firm locates final-good assembly plants (i.e., its global assembly strategy $\mathcal{K}(\varphi) \subseteq J$). Starting from equation (4), using the optimal constant markup rule (see Appendix A), and aggregating across export platforms and their destination markets, firm profits (net of the initial entry cost) can be expressed as:

(6)
$$\pi(\varphi) = \kappa_\pi \varphi^{\sigma-1} \sum_{i\in J} \left(\Psi_i(\varphi)\right)^{\frac{\sigma-1}{\varepsilon-1}} P_i^{\sigma-1} E_i - \sum_{k\in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

where κ_π is a constant and where $\Psi_i(\varphi)$ is defined in (5). Solving for the set $\mathcal{K}(\varphi)$ that maximizes equation (6) is a combinatorial problem, but regardless of its specific solution, we can characterize whether the firm's global assembly location decisions are complements or substitutes.

To build intuition, note that whenever the (cross-firm) elasticity of demand for the MNE's goods is low relative to the elasticity of within-firm labor substitution across the MNE's plants, the profitability of setting up an export platform in country k is reduced by the existence of assembly plants in other locations $k' \neq k$ because these other assembly plants cannibalize on plant k's sales (as shown in equation (4)). Similarly, setting up a new plant in k is less desirable, because this new plant would cannibalize sales from existing plants in other locations. These are canonical features of export-platform FDI models; however, equation (6) demonstrates that when within-firm labor substitution is low, the opposite may be true, and extensive margin assembly decisions are complements.

Formally, we consider an idiosyncratic increase from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$ in a given plant k's assembly potential. Denote the optimal assembly decisions under ξ_k^a and $\hat{\xi}_k^a$ by $\mathcal{I}^a = (\mathcal{I}_1^a, \ldots, \mathcal{I}_J^a)$ and $\hat{\mathcal{I}}^a = (\hat{\mathcal{I}}_1^a, \ldots, \hat{\mathcal{I}}_J^a)$, respectively. Denote by X_{-k} the vector X excluding element k. For vectors X and Y, we say that $X \geq Y$ if $X_i \geq Y_i$ for all i, and $X > Y$ if $X \geq Y$ and $X_j > Y_j$ for some j. Given this notation, we prove in Appendix A that:

PROPOSITION 1: *An increase in the assembly potential of a given plant k from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$ leads to $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ whenever $\varepsilon \leq \sigma$, but it would not lead to $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ whenever $\varepsilon > \sigma$ and $\mathcal{I}^a$ is a unique solution.*

In sum, whenever $\varepsilon > \sigma$, this baseline model cannot feature complementarities in the extensive margin of global assembly.

II. Beyond Armington

The assumption that goods are differentiated based on their country of production might be unpalatable. If instead MNEs produced a single homogeneous good, its various candidate locations would be perfect substitutes, and cannibalization effects would always dominate. Nevertheless, the vast majority of MNEs are multi-product firms, and a non-trivial part of their operational decisions relate to the optimal allocation of products to plants, taking into account each plant's productivity in the production of the firm's various goods, and their relative distance to consumers.

In Appendix A, we build on Tintelnot (2017) and develop a version of our model in which goods are *not* differentiated based on where they are produced. Instead, productivity heterogeneity across a continuum of goods generates a well-defined (and interior) allocation of products to plants. When such productivity dispersion follows a Fréchet distribution with shape parameter θ, we obtain an isomorphic set of equilibrium equations (4)–(6) with θ replacing $\varepsilon-1$ throughout. In such a case, the Fréchet parameter θ governs the substitutability of labor across an MNE's plants.

Even without imposing a Fréchet distribution of productivity, we show in Appendix A that whether assembly plants are substitutes or complements depends on the relative size of the elasticity of demand for the MNE's goods and the (Allen) *within-firm* elasticity of substitution of labor across the MNE's active plants.[1]

III. Export-Platform FDI Model with Firm-Level Export Costs

We now assume that firms incur fixed marketing costs of f_i^x units of labor in country i to sell their varieties in country i. We use the superscript x to denote these fixed costs, and assume they are incurred at the firm, rather than the plant level. Crucially, when a firm pays the fixed marketing cost to sell in country i, *all* its assembly plants may access that market. This assumption aligns well with the fact that multinational firms often centralize their sales and marketing decisions in a specialized division. We denote the optimal set of countries $i \in J$ for which a firm with productivity φ has paid the associated fixed cost of marketing by $\Upsilon(\varphi) \subseteq J$, and refer to it as the firm's *global marketing strategy.*

It should be clear that these firm-level fixed costs to export have no bearing on equation (4) capturing sales of an assembly plant in k to each market i, except that the equation now only applies to destination markets i in the *firm*'s global marketing strategy (i.e., $i \in \Upsilon(\varphi)$). Holding the firm's extensive-margin strategies constant, whether an idiosyncratic increase in the assembly potential ξ_k^a of plant k increases or decreases sales of plants based in $k' \neq k$ continues to depend only on the relative size of σ and ε, with $\varepsilon > \sigma$ leading to cannibalization and $\varepsilon < \sigma$ leading to complementarity.

Profits net of entry costs are also given by an expression almost identical to that in equation (6), namely:

$$\pi(\varphi) = \kappa_\pi \varphi^{\sigma-1} \sum_{i \in J} \mathcal{I}_i^x \cdot (\Psi_i(\varphi))^{\frac{\sigma-1}{\varepsilon-1}} P_i^{\sigma-1} E_i - \sum_{i \in J} \mathcal{I}_i^x \cdot w_i f_i^x - \sum_{k \in J} \mathcal{I}_k^a \cdot w_k f_k^a. \tag{7}$$

Despite these strong similarities with our starting model of export-platform FDI, the presence of firm-level fixed costs of exporting carries important implications for the nature of the interdependencies across the assembly plants of a firm. More specifically, denote by $\mathcal{I}^x$ and $\hat{\mathcal{I}}^x$ the optimal exporting decisions under ξ_k^a and $\hat{\xi}_k^a$, respectively. We show in Appendix A that:

PROPOSITION 2: *With firm-level fixed costs of exporting, an increase in the assembly potential of a given plant k from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$ leads to $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ and $\hat{\mathcal{I}}^x \geq \mathcal{I}^x$ whenever $\varepsilon \leq \sigma$, and it may lead to $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ and $\hat{\mathcal{I}}^x \geq \mathcal{I}^x$ even when $\varepsilon > \sigma$.*

This result implies that the model generates complementarities across assembly lo-

[1] It is important to stress that what is relevant is the *intensive-margin* elasticity of labor substitution across plants, taking the location of all plants (and their associated fixed costs) as given.

cations for a wider range of parameter values than our baseline model.

The intuition for this result is as follows. An increase in ξ_k^a necessarily increases the profits associated with sales emanating from that plant k. This increase in profitability may lead firm φ to activate export destinations that were not profitable before the increase in ξ_k^a. Crucially, because plants in other potential assembly locations $k' \neq k$ would benefit from the activation of such an export destination, this induced change in the firm-level extensive margin of exports may well increase the profitability of activating these other potential assembly locations k', especially when cannibalization effects are small.

As we show in Appendix A, the fact that fixed costs of exporting are incurred at the firm-level is crucial for these results: if these fixed costs were incurred at the plant-level, we would revert to the result stated in Proposition 1, and complementarities could not arise when $\varepsilon > \sigma$.

IV. Export-Platform FDI Model with Firm-Level Sourcing Costs

We finally relax the assumption that final goods are only produced with labor and introduce tradable intermediate inputs. Following our approach for preferences, we assume that inputs sourced from different countries are imperfect substitutes, with a constant elasticity of substitution $\rho > 1$. Intermediates are produced worldwide by a competitive fringe of suppliers that sells its products at marginal cost. All intermediates are produced with labor under a linear technology delivering Z_j^s units of output per unit of labor. Shipping intermediates from country j to country k entails iceberg trade costs τ_{jk}^s. As a result, the cost at which firms producing in k can procure inputs from country j is given by $\tau_{jk}^s w_j / Z_j^s$.

A firm must incur a country-specific fixed cost $w_j f_j^s$ to source inputs from a particular country j. Although this assumption is similar to Antràs et al. (2017), a crucial distinction here is that the fixed cost grants *all* of the firm's assembly plants $k \in \mathcal{K}(\varphi)$ access to inputs from that country. We denote the set of countries for which firm φ has paid the fixed costs of sourcing by $\mathcal{J}(\varphi) \subseteq J$ and refer to it as the firm's *global sourcing strategy*.

The overall marginal cost for firm φ to produce units of the final-good variety in country k is given by

$$c(\varphi, k) = \frac{(\xi_k^a)^{\frac{1-\alpha}{1-\varepsilon}} \left(\sum_{j \in \mathcal{J}(\varphi)} \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho} \right)^{\frac{\alpha}{1-\rho}}}{\varphi},$$

where $1-\alpha$ is the labor share in final-good production, $\xi_k^a \equiv \left(w_k / Z_k^a\right)^{1-\varepsilon}$ is the firm's assembly potential in country k, and $\xi_j^s \equiv \left(w_j / Z_j^s\right)^{1-\rho}$ is country j's *sourcing potential.*

Invoking constant-markup pricing, one can show that the sales of an assembly plant in k to each market i (we ignore fixed costs of exporting in this section) are given by

$$S_{ki}(\varphi) = \kappa \varphi^{\sigma-1} \left(\xi_k^a\right)^{1-\alpha} \left(\tau_{ki}^a\right)^{1-\varepsilon} \times \left(\Theta_k(\varphi)\right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}} \left(\Lambda_i(\varphi)\right)^{\frac{\sigma-\varepsilon}{\varepsilon-1}} E_i P_i^{\sigma-1}, \tag{8}$$

where κ is a constant. The term $\Theta_k(\varphi)$ is plant k's *sourcing capability* (see Antràs et al., 2017), and is given by

$$\Theta_k(\varphi) \equiv \sum_{j \in J} \mathcal{I}_j^s \cdot \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho},$$

where $\mathcal{I}_j^s$ is an indicator function that takes a value of 1 if the firm activates country j as a source of inputs, and 0 otherwise. Finally, the term $\Lambda_i(\varphi)$ is

$$\Lambda_i(\varphi) \equiv \sum_{k' \in J} \mathcal{I}_{k'}^a \cdot \left(\xi_{k'}^a\right)^{1-\alpha} \times \left(\tau_{k'i}^a\right)^{1-\varepsilon} \left(\Theta_{k'}(\varphi)\right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}}.$$

The empirical complementarities in global sourcing documented in Antràs et al. (2017) lead us to impose:

Assumption 1: $\alpha(\varepsilon-1) \geq \rho - 1$.

Equation (8) is significantly more involved than its counterpart (4) in our baseline model, but holding the firm's extensive-margin strategies constant, whether an increase in the assembly potential ξ_k^a of plant k increases or decreases sales of plants based in $k' \neq k$ continues to be shaped solely by the relative size of σ and ε, with $\varepsilon > \sigma$ leading to cannibalization and $\varepsilon < \sigma$ leading to complementarity.

Profits are now given by

$$\pi(\varphi) = \kappa_\pi \varphi^{\sigma-1} \sum_{i \in J} E_i P_i^{\sigma-1} \times (\Lambda_i(\varphi))^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{k \in J} \mathcal{I}_k^a \cdot w_k f_k^a - \sum_{j \in J} \mathcal{I}_j^s \cdot w_j f_j^s,$$

for some constant κ_π.

Whenever $\varepsilon \leq \sigma$, profits continue to feature increasing differences in $(\mathcal{I}_k^a, \mathcal{I}_{k'}^a)$ for $k, k' \in \{1, ..., J\}$ and $k \neq k'$. Denote by $\mathcal{I}^s$ and $\hat{\mathcal{I}}^s$ the optimal sourcing decisions under ξ_k^a and $\hat{\xi}_k^a$, respectively. In Appendix A, we show that:

PROPOSITION 3: *With firm-level fixed costs of sourcing, under Assumption 1, an increase in the assembly potential of a given plant k from ξ_k^a to $\hat{\xi}_k^a > \xi_k^a$ leads to $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$ and $\hat{\mathcal{I}}^s \geq \mathcal{I}^s$ whenever $\varepsilon \leq \sigma$, and it may lead to $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ and $\hat{\mathcal{I}}^s \geq \mathcal{I}^s$ even when $\varepsilon > \sigma$.*

As in the case of firm-level fixed export costs, the presence of firm-level fixed costs of sourcing again widens the range of parameter values for which assembly locations are complements. Intuitively, an increase in ξ_k^a increases the profitability of plant k, which in turn increases the marginal benefit of investing in a larger sourcing capability $\Theta_k(\varphi)$ for that plant. Since all of the firm's plants now benefit from a new input source country, their sourcing capabilities $\Theta_{k'}(\varphi)$ tend to be enhanced, and necessarily so under Assumption 1. This larger sourcing capability may in turn increase the profitability of activating other potential assembly locations k', especially when cannibalization effects are small. If fixed costs of sourcing were incurred at the plant- rather than the firm-level, this complementarity force would disappear, and a result analogous to the one in Proposition 1 would apply (see Appendix A).

V. Conclusion

We demonstrate that the various plants of an MNE do not cannibalize on each other whenever the elasticity of demand for the MNE's goods is low relative to the within-firm elasticity of labor substitution across the MNE's plants. We also show that this complementarity is enhanced by firm-country-specific fixed costs to sell those goods or source inputs for their production.

The existence of complementarities in the export-platform strategies of MNEs is important, among other reasons, because it constitutes a potential force contributing to the remarkable predominance of a small number of multinational firms in world trade flows, as documented for the case of the US in Antràs et al. (forthcoming).

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Online Appendix

In this Appendix, we present details that were omitted from the main text. We provide proofs for the three Propositions in the paper, and for other results claimed (without proof) in the main text.

A1. Formal Definition of Price Indexes

Denoting by $p_i(\varphi, k)$ the price charged for variety k, the overall price index $\boldsymbol{p}_i(\varphi)$ for varieties sold by firm φ is given by

$$\boldsymbol{p}_i(\varphi) = \left(\sum_{k \in \mathcal{K}} p_i(\varphi, k)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{A1}$$

The economy-wide ideal price index is in turn given by

$$P_i = \left(\int_{\varphi \in \Omega_i} \boldsymbol{p}_i(\varphi)^{1-\sigma} \mathrm{d}\varphi \right)^{\frac{1}{1-\sigma}}. \tag{A2}$$

A2. Optimal Prices

In this Appendix, we show that firms have an incentive to charge a constant markup over marginal cost for its goods, with the markup being governed by the cross-firm demand elasticity σ.

To simplify matters, we assume, without loss of generality, that $P_i^{\sigma-1} E_i = 1$. Because we focus throughout on a firm-level problem, we often omit φ subscripts in variables that are firm-specific, to make the notation a bit less cumbersome.

A firm solves the following problem in each market i:

$$\begin{array}{ll} \max\limits_{q_i(k)} & \sum\limits_{k \in \mathcal{K}} (p_i(k) - c_i(k)) \cdot q_i(k) \\ \text{s.t.} & \quad q_i(k) = p_i(k)^{-\varepsilon} \boldsymbol{p}_i^{\varepsilon-\sigma} \end{array} \tag{A3}$$

where $\mathcal{K}$ is the set of active assembly plants, $c_i(k)$ is the marginal cost of production from plant k when selling to market i, and

$$\boldsymbol{p}_i = \left(\sum_{k \in \mathcal{K}} p_i(k)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}},$$

as indicated in equation (A1). The constraint in (A3) can easily be derived from equation (3) after setting $P_i^{\sigma-1} E_i = 1$.

It is straightforward to verify that:

$$\sum_{k \in \mathcal{K}} p_i(k) \cdot q_i(k) = \boldsymbol{p}_i \cdot \boldsymbol{q}_i \text{ where } \boldsymbol{q}_i \equiv \left(\sum_{k \in \mathcal{K}} q_i(k)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} = \boldsymbol{p}_i^{-\sigma}.$$

Therefore, problem (A3) can be written as a one-dimensional profit maximization

$$\max_{\boldsymbol{q}_i} \quad \boldsymbol{q}_i^{1-\frac{1}{\sigma}} - \boldsymbol{c}_i \cdot \boldsymbol{q}_i, \tag{A4}$$

where the marginal cost $\boldsymbol{c}_i$ for producing a bundle $\boldsymbol{q}_i$ is obtained from cost minimization:

$$\begin{aligned} \boldsymbol{c}_i &= \min_{q_i(k)} \sum_{k\in\mathcal{K}} c_i(k)\cdot q_i(k) \\ \text{s.t.} & \left(\sum_{k\in\mathcal{K}} q_i(k)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1. \end{aligned} \tag{A5}$$

Solving (A4) and (A5), and substituting optimal $\boldsymbol{q}_i$ and $\{q_i(k)\}_{k\in\mathcal{K}}$ into the demand equations in (A3) gives the following optimal prices

$$p_i(k) = \frac{\sigma}{\sigma-1} c_i(k) \text{ and } \boldsymbol{p}_i = \frac{\sigma}{\sigma-1}\cdot\left(\sum_{k\in\mathcal{K}} c_i(k)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \tag{A6}$$

which are a constant markup $\sigma/(\sigma-1)$ over marginal cost.

A3. Expressions in the Main Text

In this Appendix, we explicitly derive the key expressions in the main text. We begin by using the optimal prices in (A6) to derive sales from plant k to market i (we again omit φ subscripts, for simplicity).

Starting with equation (3) in the main text, we obtain:

$$S_{ki} = p_i(k)^{1-\varepsilon}\boldsymbol{p}_i^{\varepsilon-\sigma} P_i^{\sigma-1} E_i = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \mathcal{I}_k^a \cdot c_i(k)^{1-\varepsilon} \cdot \left(\sum_{k\in J} \mathcal{I}_k^a \cdot c_i(k)^{1-\varepsilon}\right)^{\frac{\sigma-\varepsilon}{\varepsilon-1}} \cdot P_i^{\sigma-1} E_i,$$

where $\mathcal{I}_k^a = 1$ if a firm paid fixed costs of assembly in location $k\in J$, and $\mathcal{I}_k^a = 0$ otherwise. For a firm with productivity φ, the marginal costs are

$$c_i(k) = \frac{1}{\varphi}\cdot\frac{w_k}{Z_k^a}\cdot\tau_{ki}^a,$$

thereby delivering the expression in equation (4) in the main text.

The overall profit for firm φ, equation (6), is

$$\pi(\varphi) = \frac{1}{\sigma}\sum_{i\in J}\sum_{k\in J} S_{ki} = \kappa_\pi \varphi^{\sigma-1}\sum_{i\in J} P_i^{\sigma-1} E_i \cdot (\Psi_i(\varphi))^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{k\in J}\mathcal{I}_k^a \cdot w_k f_k^a,$$

where $\kappa_\pi = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$, $\mathcal{I}_k^a = 1$ if $k\in\mathcal{K}(\varphi)$, and

$$\Psi_i(\varphi) = \sum_{k\in J}\mathcal{I}_k^a\cdot\xi_k^a\left(\tau_{ki}^a\right)^{1-\varepsilon}.$$

With firm-level fixed costs of exporting, the profit function, equation (7), is

$$\pi(\varphi)=\kappa_\pi\varphi^{\sigma-1}\sum_{i\in J}\mathcal{I}_i^x\cdot P_i^{\sigma-1}E_i\cdot(\Psi_i(\varphi))^{\frac{\sigma-1}{\varepsilon-1}}-\sum_{i\in J}\mathcal{I}_i^x\cdot w_i f_i^x-\sum_{k\in J}\mathcal{I}_k^a\cdot w_k f_k^a.$$

In section IV of the main text, we introduce tradable intermediate inputs. Formally, we assume that firm φ has the following production

$$F_\varphi(\ell,Q_s)=\frac{\varphi}{(1-\alpha)^{1-\alpha}\alpha^\alpha}\ell^{1-\alpha}Q_s^{1-\alpha},$$

where ℓ is labor, and Q_s is a bundle of inputs

$$Q_s=\left(\sum_{j\in J}\mathcal{I}_j^s\cdot\left(q_j^s\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}\quad\text{where } \mathcal{I}_j^s=1\text{ if } j\in\mathcal{J}(\varphi)\text{ and }\rho>1.$$

This production function has the following marginal costs

$$c_i(\varphi,k)=\frac{1}{\varphi}\cdot\left(\frac{w_k}{Z_k^a}\right)^{1-\alpha}\cdot\left(\sum_{j\in J}\mathcal{I}_j^s\cdot\left(\frac{w_j\tau_{jk}^s}{Z_j^s}\right)^{1-\rho}\right)^{\frac{\alpha}{1-\rho}}.$$

Substituting these marginal costs into the optimal prices in (A6) we get the sales from plant k to market i, written in equation (8) in the main text.

Finally, the profit function with intermediate inputs can be written as

$$\pi(\varphi)=\kappa_\pi\varphi^{\sigma-1}\sum_{i\in J}P_i^{\sigma-1}E_i\cdot\Lambda_i(\varphi)-\sum_{j\in J}\mathcal{I}_j^s\cdot w_j f_j^s-\sum_{k\in J}\mathcal{I}_k^a\cdot w_k f_k^a,$$

where

$$\Lambda_i(\varphi)=\left[\sum_{k\in J}\mathcal{I}_k^a\cdot\left(\xi_k^a\right)^{1-\alpha}\left(\tau_{ki}^a\right)^{1-\varepsilon}\cdot\left(\sum_{j\in J}\mathcal{I}_j^s\cdot\xi_j^s\left(\tau_{jk}^s\right)^{1-\rho}\right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}}\right]^{\frac{\sigma-1}{\varepsilon-1}}$$

and

$$\xi_k^a=\left(\frac{w_k}{Z_k^a}\right)^{1-\varepsilon}\text{ and }\xi_j^s=\left(\frac{w_j}{Z_j^s}\right)^{1-\rho}.$$

A4. Relaxing the Armington Assumption

In section II of the main text, we argue that our main results are not dependent on the Armington assumption implicit in equation (2). We prove this claim in this Appendix.

Labor Substitutability in the Armington Model

We first demonstrate that, in our baseline model, ε corresponds to the within-firm elasticity of labor substitution across an MNE's plants. In that model, when figuring out the optimal way to allocate labor across plants to sell goods in market i, for a given assembly strategy $\mathcal{K}$, the firm solves the following problem

$$c_i = \min_{\{\ell_{k,i}(\nu)\}} \sum_{k\in\mathcal{K}} w_k \ell_{k,i}$$

$$\text{s.t. } \left(\sum_{k\in\mathcal{K}(\varphi)} q_i\left(k\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1 \qquad \text{(bundle of products)}$$

$$\text{s.t. } q_i(k) = \frac{Z_k^a}{\tau_{ki}^a}\cdot \ell_{k,i} \qquad \text{(production technology).}$$

The solution to this problem delivers the following cost function

$$c_i = \left(\sum_{k\in\mathcal{K}(\varphi)} \left(\tau_{ki}^a \frac{w_k}{Z_k^a}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{A7}$$

Define the conditional elasticity of labor demand in location k to changes in location l as

$$\mathcal{E}_{k,l}^i = \frac{\partial \ell_{k,i}}{\partial w_l}\frac{w_l}{\ell_{k,i}},$$

and define the share of variable labor costs associated with selling goods to i paid to labor in location l as:

$$S_l^i = \frac{w_l \ell_{l,i}}{c_i} = \frac{w_l \ell_{l,i}}{\sum\limits_{k\in\mathcal{K}} w_k \ell_{k,i}}.$$

The Allen partial elasticity of substitution is defined as

$$\varepsilon_{k,l}^i \equiv \frac{\mathcal{E}_{k,l}^i}{S_l^i}.$$

For our CES-Armington cost function in (A7), we can invoke Shephard's lemma to find:

$$\ell_{k,i} = \frac{\partial c_i}{\partial w_k} = \left(c_i\right)^{\varepsilon}\left(\frac{\tau_{ki}^a}{Z_k^a}\right)^{1-\varepsilon}\left(w_k\right)^{-\varepsilon}. \tag{A8}$$

The conditional elasticity of labor demand in location k to changes in location l is thus

$$\mathcal{E}_{k,l}^i = \frac{\partial \ell_{k,i}}{\partial w_l}\frac{w_l}{\ell_{k,i}} = \left(\frac{\tau_{ki}^a}{Z_k^a}\right)^{1-\varepsilon}\varepsilon\left(c_i\right)^{\varepsilon-1}\frac{\partial c_i}{\partial w_l}\left(w_k\right)^{-\varepsilon}\frac{w_l}{\ell_{k,i}}$$

Invoking Shephard's lemma and plugging in (A8) delivers

$$\mathcal{E}_{k,l}^i = \varepsilon \frac{w_l \ell_{l,i}}{c_i},$$

so the Allen partial elasticity of labor substitution across plants is

$$\varepsilon_{k,l}^i \equiv \frac{\mathcal{E}_{k,l}^i}{S_l^i} = \varepsilon.$$

It is also simple to see from equation (A8) that, for two locations k and l,

$$\frac{\ell_{k,i}}{\ell_{l,i}} = (c_i)^{\varepsilon} \left(\frac{\tau^a_{ki}/Z^a_k}{\tau^a_{li}/Z^a_l}\right)^{1-\varepsilon} \left(\frac{w_k}{w_l}\right)^{-\varepsilon}$$

and thus ε also corresponds to the more traditional Hicks elasticity of substitution, defined as

$$\tilde{\mathcal{E}}^i_{k,l} = \frac{\partial \ln \left(\ell_{k,i}/\ell_{l,i}\right)}{\partial \ln \left(w_l/w_k\right)}.$$

It is important to stress that ε measures the *intensive-margin* elasticity of labor substitution, taking as fixed the location of the various plants and without consideration to the labor investments that might have been incurred when setting up those plants.

Labor Substitutability with Productivity differences à la Eaton-Kortum

We next explore the robustness of our results to a version of our model in which goods are *not* differentiated based on where they are produced. This version constitutes a simple extension of the model in Tintelnot (2017).

There is an endogenous measure Ω_i of manufacturing firms selling goods in country i. As in Tintelnot (2017), each of these firms produces and sells a continuum of measure one of varieties of manufactured goods. We continue to index firms by φ and varieties within firms by ω. We assume a nested-CES structure in which the degree of substitutability σ across varieties produced by different firms, and the degree of substitutability σ_w across varieties produced by the same firm may differ from each other:

$$U_{Mi} = \left(\int\limits_{\varphi\in\Omega_i} \left(\int_0^1 q_i\left(\varphi,\omega\right)^{(\sigma_w-1)/\sigma_w}\,\mathrm{d}\omega\right)^{\frac{\sigma_w}{\sigma_w-1}\frac{(\sigma-1)}{\sigma}} \mathrm{d}\varphi \right)^{\sigma/(\sigma-1)}, \quad \sigma_w, \sigma > 1.$$

These preferences imply that consumers in country i spend a share

$$s_i(\varphi) = \left(\frac{p_i(\varphi)}{P_i}\right)^{1-\sigma} E_i \tag{A9}$$

of their income on firm φ. In this expression, E_i is total spending on manufactured goods in country $i \in J$,

$$\boldsymbol{p}_i(\varphi) = \left(\int\limits_0^1 p_i(\varphi,\omega)^{1-\sigma_w} \mathrm{d}v \right)^{\frac{1}{1-\sigma_w}} \tag{A10}$$

is the overall price index for varieties sold by firm φ, and P_i is the economy-wide ideal price index in country i (given again by equation (A2)). Note that, as in our baseline model, σ continues to govern the cross-firm elasticity of demand faced by firm φ.

On the production side, we let firms produce their continuum of products in multiple countries. Given fixed costs of assembly (identical to those in our baseline model), firms will typically produce only in a subset of all countries in the world, and we denote this set $\mathcal{K} \subseteq J$ as the firm's *global assembly strategy.* Shipping final goods from country k to country i entails variable (iceberg) trade costs τ^a_{ki}. In line with our baseline model and with Tintelnot (2017), we abstract from fixed costs of exporting.

The marginal cost for firm φ to produce units of final-good variety ω in country k is given by

$$c(\varphi, k, \omega) = \frac{1}{\varphi} \frac{1}{z_k(\varphi, \omega)} w_k, \tag{A11}$$

where $z_k(\varphi, \omega)$ is a firm- and location-specific labor productivity term. Following Tintelnot (2017), we assume that these firm- and location-specific assembly productivity shifters are drawn from the following Fréchet distribution:

$$\Pr(1/z_k(\varphi, \omega) \geq a) = e^{-(Z_k^a a)^\theta}, \quad \text{with } Z_k^a > 0. \tag{A12}$$

Z_k^a governs the average productivity of plant k, while θ determines the dispersion of productivity draws across final-good varieties, with a lower θ indicating a higher variance, and thus greater benefits from producing final-good varieties in various locations. To ensure a well-defined solution, we follow Tintelnot (2017) in imposing a lower bound on the dispersion in the final-good productivity draws $z_k(\varphi, \omega)$:

Technical Assumption: $\sigma_\omega - 1 < \theta$.

Following the derivations in Tintelnot (2017), it is possible to show that this Eaton-Kortum formulation results in a marginal cost for firm φ of selling its bundle of goods to market i, which is given by

$$c_i(\varphi) = \kappa \cdot \left(\sum_{k \in \mathcal{K}(\varphi)} \left(\tau_{ki}^a \frac{w_k}{Z_k^a} \right)^{-\theta} \right)^{-1/\theta}, \tag{A13}$$

where κ is a constant. As claimed in the main text, this marginal cost is identical (up to a constant) to that in equation (A7), with θ replacing $\varepsilon - 1$. Because firms charge a constant markup $\sigma / (\sigma - 1)$ over this marginal cost, the rest of the equilibrium conditions of this version of our model, i.e., the analogues of equations (4)–(6), are identical to those in the main text with θ replacing $\varepsilon - 1$. The isomorphism between (A7) and (A13) also makes it clear that the Allen partial elasticity of labor substitution across plants is now given by $\theta + 1$, and whether assembly locations are complements or substitutes depends on the relative size of the (cross-firm) demand elasticity σ and this labor substitution elasticity $\theta + 1$.

It is also worth pointing out that Tintelnot (2017) focused on symmetric CES preferences with a common degree of substitutability across varieties produced by different firms and across varieties produced by the same firm, or $\sigma = \sigma_\omega$. The technical assumption $\sigma_\omega - 1 < \theta$ then led him to assume $\sigma - 1 < \theta$, which implies that assembly locations were necessarily substitutes in his framework. But if $\sigma_\omega < \sigma$, under our more general nested CES structure, it is perfectly possible for assembly locations to be complements ($\sigma - 1 > \theta$) while ensuring a well-defined firm-level problem ($\sigma_\omega - 1 < \theta$).

A More General Production Structure

We finally consider a more general production structure that encompasses to two models developed above and more general settings. We focus on the problem of a firm that produces a set of varieties $\mathcal{V}$ (for simplicity we drop firm-specific subscripts). For each destination $i \in J$, varieties are bundled according to

$$Q_i = F_i\left(\{q_i(\nu)\}_{\nu \in \mathcal{V}}\right),$$

and consumers have CES preferences over Q_i across firms, with elasticity of substitution σ. Each variety is produced using labor from different locations in the firm's global assembly strategy according to

$$q_i(\nu) = F_i^{\nu}\left(\{\ell_{k,i}(\nu)\}_{k\in\mathcal{K}}\right).$$

The operating profit function (excluding fixed costs) can be written as

$$\pi^o = \kappa \cdot \sum_{i\in J} c_i^{1-\sigma} \cdot P_i^{\sigma-1} E_i$$

where c_i is the marginal cost of producing a bundle of goods to be sold in destination i. These marginal costs come from a cost-minimization problem:

$$\begin{aligned}
c_i = \min_{\{\ell_{k,i}(\nu)\}} & \sum_{\nu\in\mathcal{V}}\sum_{k\in\mathcal{K}} w_k \ell_{k,i}(\nu) \\
\text{s.t. } & F_i\left(\{q_i(\nu)\}_{\nu\in\mathcal{V}}\right) = 1 && \text{(bundle of products)} \\
\text{s.t. } & q_i(\nu) = F_i^{\nu}\left((\ell_{k,i}(\nu))_{k\in\mathcal{K}}\right) && \text{(production technology)}
\end{aligned}$$

We shall say that assembly locations are (local) substitutes if $\frac{\partial^2 \pi^o}{\partial w_k \partial w_l} < 0$ and (local) complements if $\frac{\partial^2 \pi^o}{\partial w_k \partial w_l} > 0$ for $k \neq l$.[2] To compute these expressions, we calculate

$$\begin{aligned}
\frac{\partial^2 c_i^{1-\sigma}}{\partial w_k \partial w_l} &= \frac{\partial}{\partial w_l}\left[(1-\sigma)c_i^{-\sigma}\cdot\frac{\partial c_i}{\partial w_k}\right] = \frac{\partial}{\partial w_l}\left[(1-\sigma)c_i^{-\sigma}\cdot \ell_{k,i}\right] = \\
&= (1-\sigma)\cdot\left[c_i^{-\sigma}\frac{\partial \ell_{k,i}}{\partial w_l} - \sigma\cdot c_i^{-\sigma-1}\ell_{l,i}\cdot\ell_{k,i}\right]
\end{aligned}$$

where we use Shephard's lemma to derive the total demand for labor from location k, $\frac{\partial c_i}{\partial w_k} = \sum_{\nu\in\mathcal{V}} \ell_{k,i}(\nu) \equiv \ell_{k,i}$, and location l, $\frac{\partial c_i}{\partial w_l} = \sum_{\nu\in\mathcal{V}} \ell_{l,i}(\nu) \equiv \ell_{l,i}$.

It thus follows that assembly locations are (local) substitutes or complements, respectively, if

$$\text{(A14)} \qquad \min_{i,l,k}\left\{\frac{\mathcal{E}_{k,l}^i}{S_l^i}\right\} > \sigma \text{ or } \max_{i,l,k}\left\{\frac{\mathcal{E}_{k,l}^i}{S_l^i}\right\} < \sigma,$$

where $\mathcal{E}_{k,l}^i$ is the elasticity of substitution of conditional demand for labor in location k with respect to the price of labor in location l, and S_l^i is share of spending on labor from l in total spending on labor from different countries to serve market i:

$$\mathcal{E}_{k,l}^i = \frac{\partial \ell_{k,i}}{\partial w_l}\frac{w_l}{\ell_{k,i}} \text{ and } S_l^i = \frac{w_l \ell_{l,i}}{c_i} = \frac{w_l \ell_{l,i}}{\sum_{k\in\mathcal{K}} w_k \ell_{k,i}}.$$

In sum, we have that assembly locations are (local) substitutes or complements, respectively, if

$$\min_{i,l,k}\left\{\varepsilon_{k,l}^i\right\} > \sigma \text{ or } \max_{i,l,k}\left\{\varepsilon_{k,l}^i\right\} < \sigma,$$

where $\varepsilon_{k,l}^i$ is the (Allen) partial elasticity of substitution of labor across locations k and l, when producing goods for sale in market i.

Special Cases. The Armington setting in the main text corresponds to the following

[2]We assume that wages are firm-specific, so the aggregate demand $P_i^{\sigma-1}E_i$ is constant.

assumptions

$$\mathcal{V} = \mathcal{K}$$
$$F_i\left(\{q_i(\nu)\}\right) = \left(\sum_{\nu\in\mathcal{K}} q_i(\nu)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$q_i(\nu) = Z^a_{k,i}\cdot \ell_{k,i}(\nu) \text{ for } \nu = k \text{ and } Z^a_{k,i} = \frac{Z^a_k}{\tau^a_{k,i}} > 0$$
$$q_i(\nu) = 0 \text{ for } \nu \neq k,$$

while the setting in Tintelnot (2017) (extended to nested CES preferences) corresponds to[3]

$$\mathcal{V} = [0,1]$$
$$F_i\left(\{q_i(\nu)\}\right) = \left(\int_0^1 q_i(\nu)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$q_i(\nu) = \sum_{k\in\mathcal{K}} Z^a_{k,i}(\nu)\cdot \ell_{k,i}(\nu).$$

A5. *Proofs of Propositions 1-3*

Notation

Consider the general problem with firm- and plant-level fixed costs. Denote by $\mathcal{I}^x_i = 1$ if a firm paid firm-level fixed costs of marketing to destination i, $w_i f^x_i$, and $\mathcal{I}^x_i = 0$ otherwise; by $\mathcal{I}^s_j = 1$ if a firm paid firm-level fixed costs of importing from sourcing location j, $w_j f^s_j$, and $\mathcal{I}^s_j = 0$ otherwise; by $\mathcal{I}^a_k = 1$ if a firm paid firm-level fixed costs of assembly in location k, $w_k f^a_k$, and $\mathcal{I}^a_k = 0$ otherwise; by $\mathcal{I}^x_{ki} = 1$ if a firm paid plant-destination specific fixed costs of exporting from plant k to destination i, $w_i f^x_{ki}$, and $\mathcal{I}^x_{ki} = 0$ otherwise; by $\mathcal{I}^s_{jk} = 1$ if a firm paid sourcing-assembly specific fixed costs of importing from sourcing location j to assembly plant k, $w_j f^s_{jk}$, and $\mathcal{I}^s_{jk} = 0$ otherwise.

We denote by $\mathcal{I}^a = (\mathcal{I}^a_1, \ldots, \mathcal{I}^a_J)$ the vector of optimal decisions for assembly locations under ξ^a, and by $\hat{\mathcal{I}}^a = (\hat{\mathcal{I}}^a_1, \ldots, \hat{\mathcal{I}}^a_J)$ the optimal solution under $\hat{\xi}^a$. In a similar way, we denote by $\mathcal{I}^x$, $\mathcal{I}^s$, $\hat{\mathcal{I}}^x$, $\hat{\mathcal{I}}^s$ the vectors of optimal decisions for exporting and sourcing. We also denote by $\mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^a_{-k}$ the vectors $\mathcal{I}^a$ and $\hat{\mathcal{I}}^a$ without elements $\mathcal{I}^a_k$ and $\hat{\mathcal{I}}^a_k$, respectively. For vectors X and Y, we say that $X \geq Y$ if $X_i \geq Y_i$ for all i, and $X > Y$ if $X \geq Y$ and $X_j > Y_j$ for some j.

In all propositions, we assume that $\xi^a_k > 0$ and $\xi^s_j > 0$ for all $k \in J$ and $j \in J$.

[3]We replaced the sum with an integral.

General Profit Function

Consider the general profit function with firm- and plant-level fixed costs:

(A15)

$$\pi = \kappa_\pi \varphi^{\sigma-1} \cdot \overbrace{\sum_{i\in J} \mathcal{I}_i^x \cdot E_i P_i^{\sigma-1}}^{\text{Destinations}} \left[\sum_{k\in J} \overbrace{\mathcal{I}_{k,i}^x \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon}}^{\text{Assembly}} \left(\overbrace{\sum_{j\in J} \mathcal{I}_{j,k}^s \mathcal{I}_j^s \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho}}^{\text{Sourcing}} \right)^{\mu} \right]^{\theta} -$$
$$- \underbrace{\sum_{i\in J}\sum_{k\in J} \mathcal{I}_{k,i}^x \cdot w_i f_{k,i}^x - \sum_{k\in J}\sum_{j\in J} \mathcal{I}_{j,k}^s \cdot w_j f_{j,k}^x}_{\text{Plant-Level FC}} - \underbrace{\sum_{i\in J} \mathcal{I}_i^x \cdot w_i f_i^x - \sum_{k\in J} \mathcal{I}_k^a \cdot w_k f_k^a - \sum_{j\in J} \mathcal{I}_j^s \cdot w_j f_j^s}_{\text{Firm-Level FC}},$$

where

$$\theta = \frac{\sigma-1}{\varepsilon-1} \text{ and } \mu = \frac{\alpha(\varepsilon-1)}{\rho-1}.$$

If $\sigma \geq \varepsilon$ and $\alpha(\varepsilon-1) \geq \rho-1$, then the profit function in (A15) is supermodular in $(\mathcal{I}', \mathcal{I}'')$ and has increasing differences in $(\mathcal{I}, \xi_k^a)$, where $\mathcal{I}'$ and $\mathcal{I}''$ are two any indicator variables in (A15). Therefore, by Topkis' Theorem

$$\text{If } \hat{\xi}_k^a \geq \xi_k^a, \text{ then } \hat{\mathcal{I}} \geq \mathcal{I}.$$

As shown below, this result will suffice to prove all Propositions for the case of $\sigma \geq \varepsilon$ and $\alpha(\varepsilon-1) \geq \rho-1$.

Proposition 1

In our baseline model without fixed costs of exporting or intermediate inputs, a firm solves the following problem:

$$\text{(A16)} \quad \max_{\mathcal{I}^a} \ \pi\left(\mathcal{I}^a; \xi^a\right) = \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i\in J} E_i P_i^{\sigma-1} \left[\sum_{k\in J} \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{k\in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

which is a special case of (A15) under $\mu = 0$ and all fixed costs equal to zero except for the assembly ones, $f_k^a > 0$. We prove the following proposition:

PROPOSITION 1: *Consider the problem in* (A16) *and an increase in the assembly potential of plant k, $\hat{\xi}_k^a > \xi_k^a$, holding other parameters and $P_i^{\sigma-1} E_i$ fixed. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$. If $\varepsilon > \sigma$ and $\mathcal{I}^a$ is a unique solution, then $\hat{\mathcal{I}}_k^a \geq \mathcal{I}_k^a$, and it is* ***not*** *possible that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$.*

PROOF:

For the case $\varepsilon \leq \sigma$, we can apply Topkis' theorem.

Consider the case $\varepsilon > \sigma$. If $\mathcal{I}^a$ is an optimal solution under $\xi^a = (\xi_1^a, \ldots, \xi_k^a, \ldots, \xi_J^a)$, then

$$\pi(\mathcal{I}^a; \xi^a) \geq \pi(\tilde{\mathcal{I}}^a; \xi^a) \text{ for all } \tilde{\mathcal{I}}^a \in 2^J.$$

To prove that $\hat{\mathcal{I}}_k^a \geq \mathcal{I}_k^a$, assume, by contradiction, that $\hat{\mathcal{I}}_k^a = 0 < \mathcal{I}_k^a = 1$. Notice that $\pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a\right)$ is increasing in ξ_k^a while $\pi\left(\mathcal{I}_k^a = 0, \tilde{\mathcal{I}}_{-k}^a; \xi^a\right)$ is independent of ξ_k^a for all $\tilde{\mathcal{I}}_{-k}^a$ and ξ_{-k}^a, where ξ_{-k}^a is vector ξ^a without an element ξ_k^a. Therefore,

$$\pi(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \hat{\xi}^a) > \pi(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a) \geq \pi(\hat{\mathcal{I}}_k^a = 0, \hat{\mathcal{I}}_{-k}^a; \xi^a) = \pi(\hat{\mathcal{I}}_k^a = 0, \hat{\mathcal{I}}_{-k}^a; \hat{\xi}^a),$$

which is a contradiction. Therefore, $\hat{\mathcal{I}}_k^a \geq \mathcal{I}_k^a$.

For the second part, suppose, by contradiction, that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$. Consider three cases. First, suppose that $\mathcal{I}_k^a = 1$. Then, $\hat{\mathcal{I}}_k^a = 1$, and

$$\text{(A17)} \quad \pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \hat{\xi}^a\right) - \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \hat{\xi}^a\right) < \pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \xi^a\right) - \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a\right) \leq 0,$$

where the the first inequality comes from $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$ and $\varepsilon > \sigma$, and the second inequality comes from the optimality of $\mathcal{I}^a$ under ξ^a. This inequality contradicts the optimality of $\hat{\mathcal{I}}^a$ under $\hat{\xi}^a$.

Second, suppose that $\hat{I}_k^a = 0$. Then, $\mathcal{I}_k^a = 0$, and $\hat{\mathcal{I}}^a$ should be the optimal solution under both ξ^a and $\hat{\xi}^a$. This result contradicts the uniqueness of the solution.

Finally, suppose that $\mathcal{I}_k^a = 0$ and $\hat{\mathcal{I}}_k^a = 1$. The optimality of $\mathcal{I}^a$ under ξ^a implies

$$\pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \xi^a\right) - \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \xi^a\right) \leq \pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \xi^a\right) - \pi\left(\mathcal{I}_k^a = 0, \mathcal{I}_{-k}^a; \xi^a\right) \leq 0.$$

Combining this inequality with (A17), we get a contradiction for the optimality of $\hat{\mathcal{I}}^a$: $\pi\left(\hat{\mathcal{I}}_k^a = 1, \hat{\mathcal{I}}_{-k}^a; \hat{\xi}^a\right) < \pi\left(\mathcal{I}_k^a = 1, \mathcal{I}_{-k}^a; \hat{\xi}^a\right)$. □

Note: If parameters in (A15) are randomly drawn from continuous distributions, the solution is generically unique. To see the problem with multiple solutions, consider the following example. There are two plant decisions and one market with $\kappa_\pi \varphi^{\sigma-1} E_i P_i^{\sigma-1} = 1$. Suppose that $w_1 f_1^a = 100$, $w_2 f_2^a = 1$, $\xi_2^a = 1$, $\tau_{1i}^a = \tau_{2i}^a = 1$, and we consider a change from $\xi_1^a = 1$ to $\hat{\xi}_2^a = 2$. The firm chooses $\mathcal{I}_1^a = \hat{\mathcal{I}}_1^a = 0$, it is indifferent between $\mathcal{I}_2^a = 1$ and $\mathcal{I}_2^a = 0$ under ξ_1^a, and between $\hat{\mathcal{I}}_2^a = 1$ and $\hat{\mathcal{I}}_2^a = 0$ under $\hat{\xi}_1^a$. Therefore, we might have $\mathcal{I}_2^a = 0$ and $\hat{\mathcal{I}}_2^a = 1$ due to multiplicity, leading to $\hat{\mathcal{I}}_{-1}^a > \mathcal{I}_{-1}^a$ for $\hat{\xi}_1^a > \xi_1^a$. If we specify a solution selection, the proposition can be refined for the case with multiple solutions, for instance, by always choosing the solution with the largest number of active plants.

Proposition 2

We add firm-level exporting fixed costs. A firm solves the following problem:

$$\text{(A18)} \quad \max_{\mathcal{I}^a, \mathcal{I}^x} \quad \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i \in J} \mathcal{I}_i^x \cdot E_i P_i^{\sigma-1} \left[\sum_{k \in J} \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon}\right]^{\frac{\sigma-1}{\varepsilon-1}} - \sum_{i \in J} \mathcal{I}_i^x \cdot w_i f_i^x - \sum_{k \in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

which is a special case of (A15) under $\mu = 0$ and all fixed costs equal to zero except assembly and firm-level exporting ones, $f_k^a > 0$ and $f_i^x > 0$. We prove the following proposition:

PROPOSITION 2: *Consider the problem with firm-level fixed costs of exporting* (A18) *and an increase in the assembly potential of plant k, $\hat{\xi}_k^a > \xi_k^a$, holding other parameters and $P_i^{\sigma-1} E_i$ fixed. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$. If $\varepsilon > \sigma$, then it is possible that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$.*

PROOF:

For the case $\varepsilon \leq \sigma$, we can apply Topkis' theorem.

Consider the case $\varepsilon > \sigma$, it is sufficient to construct an example in which a rise in ξ_k^a leads to an opening of assembly plants in $l \neq k$. For simplicity, we assume that there is only one feasible destination market i, with $\tau_{ki'}^a = \infty$ for $i' \neq i$. Suppose that all assembly fixed costs are very small and equal to $\delta > 0$. The firm-level fixed cost of exporting to i is such that

$$\kappa_\pi \varphi^{\sigma-1} \cdot E_i P_i^{\sigma-1} \cdot \left(\xi_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} < w_i f_i^x$$

$$\kappa_\pi \varphi^{\sigma-1} \cdot E_i P_i^{\sigma-1} \cdot \left(\hat{\xi}_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} > w_i f_i^x.$$

For sufficiently small δ, an increase in ξ_k^a leads from an optimum with no assembly plants to the optimum in which all plants are activated. □

Proposition 3

We add firm-level importing fixed costs. A firm solves the following problem:

(A19)

$$\max_{\mathcal{I}^a, \mathcal{I}^s} \quad \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i \in J} E_i P_i^{\sigma-1} \left[\sum_{k \in J} \mathcal{I}_k^a \cdot \xi_k^a \left(\tau_{ki}^a\right)^{1-\varepsilon} \cdot \left(\sum_{j \in J} \mathcal{I}_j^s \cdot \xi_j^s \left(\tau_{jk}^s\right)^{1-\rho} \right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \\ - \sum_{j \in J} \mathcal{I}_j^x \cdot w_j f_j^s - \sum_{k \in J} \mathcal{I}_k^a \cdot w_k f_k^a,$$

which is a special case of (A15) under $\mu > 0$ and all fixed costs equal to zero except assembly and firm-level importing ones, $f_k^a > 0$ and $f_j^s > 0$. We prove the following proposition:

PROPOSITION 3: *Consider the problem with firm-level fixed costs of importing* (A19) *and an increase in the assembly potential of plant k, $\hat{\xi}_k^a > \xi_k^a$, holding other parameters and $P_i^{\sigma-1} E_i$ fixed. If $\varepsilon \leq \sigma$, then $\hat{\mathcal{I}}^a \geq \mathcal{I}^a$. Assume that $\alpha(\varepsilon-1) \geq \rho - 1$. If $\varepsilon > \sigma$, then it is possible that $\hat{\mathcal{I}}_{-k}^a > \mathcal{I}_{-k}^a$.*

PROOF:

Consider the following example. Assume that there is only one feasible destination market i, with $\tau_{ki'}^a = \infty$ for $i' \neq i$, and one sourcing location j, with $\tau_{j'k}^s = \infty$ for $j' \neq j$. Assume also that $\tau_{jk}^s = 1$ for all k. Suppose that all assembly fixed costs are very small and equal to $\delta > 0$. The firm-level fixed cost of sourcing from j is such that

$$\kappa_\pi \varphi^{\sigma-1} E_i P_i^{\sigma-1} \cdot \left(\xi_j^s\right)^{\frac{\alpha(\sigma-1)}{\rho-1}} \cdot \left(\xi_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} < w_j f_j^s$$

$$\kappa_\pi \varphi^{\sigma-1} E_i P_i^{\sigma-1} \cdot \left(\xi_j^s\right)^{\frac{\alpha(\sigma-1)}{\rho-1}} \cdot \left(\hat{\xi}_k^a \left(\tau_{ki}\right)^{1-\varepsilon} + \sum_{l \neq k} \xi_l^a \left(\tau_{li}^a\right)^{1-\varepsilon} \right)^{\frac{\sigma-1}{\varepsilon-1}} > w_j f_j^s.$$

For sufficiently small δ, an increase in ξ_k^a leads from an optimum with no assembly plants to the optimum in which all plants are activated. □

Plant-Level Fixed Costs

Consider the problem with plant-level fixed costs of exporting. A firm solves the following problem:

$$\begin{aligned}\text{(A20)} \quad \max_{\mathcal{I}^x,\mathcal{I}^a} \quad & \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i \in J} E_i P_i^{\sigma-1} \left[\sum_{k \in J} \mathcal{I}^x_{k,i} \mathcal{I}^a_k \cdot \xi^a_k \left(\tau^a_{ki}\right)^{1-\varepsilon} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \\ & - \underbrace{\sum_{i \in J} \sum_{k \in J} \mathcal{I}^x_{k,i} \cdot w_i f^x_{k,i}}_{\text{Plant-Level FC}} - \underbrace{\sum_{k \in J} \mathcal{I}^a_k \cdot w_k f^a_k}_{\text{Firm-Level FC}} .\end{aligned}$$

We can then prove that:

PROPOSITION 4: *Consider the problem in* (A20) *and an increase in the assembly potential of plant k,* $\hat{\xi}^a_k > \xi^a_k$*, holding other parameters and* $P_i^{\sigma-1} E_i$ *fixed. If* $\varepsilon \le \sigma$*, then* $\hat{\mathcal{I}}^a \ge \mathcal{I}^a$ *and* $\hat{\mathcal{I}}^x \ge \mathcal{I}^x$*. If* $\varepsilon > \sigma$ *and the solution is unique, then* $\hat{\mathcal{I}}^a_k \ge \mathcal{I}^a_k$*, and it is **not** possible that* $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ *and* $\hat{\mathcal{I}}^x > \mathcal{I}^x$*.*

PROOF:
For the case $\varepsilon \le \sigma$, we can apply Topkis' theorem. Consider the case $\varepsilon > \sigma$. The proof follows the same steps as the proof of Proposition 1. Under $\varepsilon > \sigma$, the assumption $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^x > \mathcal{I}^x$ contradicts the optimality (or uniqueness) of the solution. □

Now consider the problem with plant-level fixed costs of importing. A firm solves the following problem:

(A21)

$$\begin{aligned}\max_{\mathcal{I}^s,\mathcal{I}^a} \quad & \kappa_\pi \varphi^{\sigma-1} \cdot \sum_{i \in J} E_i P_i^{\sigma-1} \left[\sum_{k \in J} \mathcal{I}^a_k \cdot \xi^a_k \left(\tau^a_{ki}\right)^{1-\varepsilon} \cdot \left(\sum_{j \in J} \mathcal{I}^s_{j,k} \xi^s_j \left(\tau^s_{jk}\right)^{1-\rho} \right)^{\frac{\alpha(\varepsilon-1)}{\rho-1}} \right]^{\frac{\sigma-1}{\varepsilon-1}} - \\ & - \underbrace{\sum_{k \in J} \sum_{j \in J} \mathcal{I}^s_{j,k} \cdot w_j f^x_{j,k}}_{\text{Plant-Level FC}} - \underbrace{\sum_{k \in J} \mathcal{I}^a_k \cdot w_k f^a_k}_{\text{Firm-Level FC}} .\end{aligned}$$

We can then prove that:

PROPOSITION 5: *Consider the problem in* (A21) *and an increase in the assembly potential of plant k,* $\hat{\xi}^a_k > \xi^a_k$*, holding other parameters and* $P_i^{\sigma-1} E_i$ *fixed. Assume that* $\alpha(\varepsilon - 1) \ge \rho - 1$*. If* $\varepsilon \le \sigma$*, then* $\hat{\mathcal{I}}^a \ge \mathcal{I}^a$ *and* $\hat{\mathcal{I}}^s \ge \mathcal{I}^s$*. If* $\varepsilon > \sigma$ *and the solution is unique, then* $\hat{\mathcal{I}}^a_k \ge \mathcal{I}^a_k$*, and it is **not** possible that* $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ *and* $\hat{\mathcal{I}}^s > \mathcal{I}^s$*.*

PROOF:
For the case $\varepsilon \le \sigma$, we can apply Topkis' theorem. Consider the case $\varepsilon > \sigma$. The proof follows the same steps as the proof of Proposition 1. Under $\varepsilon > \sigma$, the assumption $\hat{\mathcal{I}}^a_{-k} > \mathcal{I}^a_{-k}$ and $\hat{\mathcal{I}}^s > \mathcal{I}^s$ contradicts the optimality (or uniqueness) of the solution. □